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Can education targets be met without increasing public spending?  
An analysis for Brazilian municipalities

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## Can education targets be met without increasing public spending? An analysis for Brazilian municipalities

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**Abstract:** The purpose of this paper is to evaluate whether the financial resources municipalities currently allocate to education are sufficient to guarantee that quality targets established for 2021 can be achieved. We use directional distance functions to determine only the proportion by which discretionary inputs should be reduced. We assume that mother's schooling is a fixed input in the short run, and that municipalities have control only over the amount of spending on education. After measuring the amount of wasted resources, we solve another linear programming problem similar to that of the DEA model, whose result is the minimum amount of spending required to achieve the 2021 quality target. Results indicate that the amount of wasted resources is significant and that targets could be achieved without additional financial resources. We also test more ambitious targets (up to 10% higher) and still we conclude that there is no scarcity of financial resources.

**Keywords:** Data Envelopment Analysis, Directional Efficiency, Public Spending, Education

**JEL Classification:** C14, H52, I28, R15

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## 1. Introduction

Municipalities enormously benefited from the federative system redesign after redemocratization in Brazil. As a consequence, municipalities increased their spending, but, at the same time, were entitled to more government transfers. With respect to education, the Constitution of 1988 mandates that municipalities concentrate their spending on early childhood, elementary, and middle school education, which eventually gave rise to the process known as municipalization of education.

At first, however, the adoption of this directive was fairly cautious, especially because other transfer programs had serious flaws, many of which occurred due to scant financial resources.

The municipalization of local elementary and middle schools only took a step further in January 1998 with the creation of FUNDEF, a fund for financing subnational spending on education.

The Constitution of 1988 established that municipalities had to allocate 25% of their revenues (taxes and transfers) to education. With the implementation of FUNDEF, municipalities transferred 15% of their revenues to the Fund (which also received transfers from states and the federal government), and later shared these revenues among municipalities proportionally, based on the number of students enrolled in the local elementary and middle schools.

The rule for revenue sharing adopted by FUNDEF, based on the number of enrolled students, strongly encouraged municipalities to proceed with the decentralization provided for in the Constitution, as it guarantees the allocation of funds therefor. By linking the availability of resources to the number of enrolled students rather than to the financial capacity of a municipality, it eventually led municipalities to stimulate school attendance by children.

Thus, the overall number of enrollments in local elementary (grade 1 through 4) schools rose from 52.7% in 1997 to 75% in 2005. For middle (grade 5 through 8) schools this increase was from 22.1% in 1997 to 41% in 2005. In 1991, there were approximately 16.7 million students in state schools and 8.7 million in local schools. In 2007, these figures went to 11.3 million and 17.6 million in state and local schools, respectively, according to Inep - Brazilian Institute for Education Studies and Research.

Transfers from the federal government to FUNDEF clearly had an equalization feature. Hence, wealthier municipalities, which spent more on a per-student basis before the creation of FUNDEF, lost financial resources from transfers to the Fund. Just to have an idea, within-state transfers (which sum zero), clearly benefitted municipalities across all regions, except for the Southeast. According to Menezes-Filho and Pazello [1], in 1998, governments of northeastern states transferred 1,810.6 million reais to FUNDEF and received R\$ 1,203.2 million reais back from the Fund. In that same year, local governments of the northeastern region transferred R\$ 966.1 million to FUNDEF and were paid R\$ 1,573.8 million reais back from the Fund.

Unquestionably, the total amount of financial resources for education in the hands of local governments did increase, and the revenue sharing pattern between states and municipalities and among municipalities in the same state eventually changed. Until 2002, municipalities received fewer resources from the Fund than did states, while the opposite is true ever since, according to Mendes, Miranda and Cossio [2].

There is ample international evidence of a weak correlation between larger resource allocation to education and school performance (see Hanushek and Kimko [3] and Hanushek and Luque [4]). For

Brazil, Amaral and Menezes-Filho [5] also conclude that the effect of spending on school performance, measured by the results of the nationwide standardized test known as *Prova Brasil*, is negligible and, in most of the adopted specifications, statistically insignificant. However, these results are based on the specification and estimation of linear models for the relationship between quality of education and its determinants.

The present paper moves away from this literature in that it is not concerned with the average, but with better practices. Input distance functions were used, but instead of reducing all inputs proportionately, we opted for directional distance functions.

In our framework, output is measured by the results of the Brazilian Education Quality Index, known as IDEB, which combines information on performance using standardized tests and on pass rates. Local government spending on elementary and middle school education and mothers' schooling are used as input. We assume that mothers' schooling is a short-run fixed input and that it lies outside a municipality's immediate discretionary power and, as such, we seek only to assess improvements in the discretionary (controllable) domain, that is, in spending itself. In other words, we investigate if it is possible to reduce spending without changing fixed input (mothers' schooling) or output levels.

The first aim of this paper is to measure each municipality's distance from the education production frontier. The resulting excess or directional distance function indicates how large is the spending on education compared to necessary resources to efficiently produce the result obtained from IDEB. A large excess ultimately means waste of resources and inefficiency.

Since its very conception, IDEB has been associated with long-term goals. The aim is to obtain education rates in 2021 that measure up to those of OECD countries, which have the best rankings worldwide in terms of quality of education.

Therefore, in addition to calculating the waste of resources, we solved a linear programming problem akin to that of the DEA model, whose result is the minimum amount of spending necessary for a municipality to achieve its 2021 IDEB target. By comparing what a municipality should spend on education if it were efficient (effective spending minus wasted resources calculated by the directional distance function) and the minimum amount of spending necessary to reach the target, it is possible to determine whether municipalities will need further resources.

To avoid a possible critique about how ambitious 2021 IDEB targets are, we increased them up to 10% and computed the minimum amount required to achieve these new targets. Results indicate that even if targets were up to 10% higher, no additional resources would be needed, as long as municipalities were efficient.

This paper is organized into five sections, besides this introduction. Section 2 provides a brief review of the international and Brazilian literature on the efficiency of spending on education. Section 3 discusses the method proposed to assess whether better resource management is enough so that municipalities reach their quality of education targets. Section 4 presents the major results. Section 5 concludes.

## 2. Literature Review

Given that education is a process that combines several inputs (students' skills, school characteristics, institutional factors, money invested in the sector, etc.) to "produce" students' competencies, the literature conceives an input/output framework to assess how well different decision-making units fare relative to each other. The concept of efficiency is then used, which "standardizes" the results (outputs) according to the inputs used in the process.

The first systematized attempt at assessing the efficiency of spending on education was made by Clements [6], who compared the relative performance of OECD countries in the delivery of educational services.

Gupta, Verhoeven and Tiongson [7] analyzed the relationship between spending and school performance for a sample of 38 African countries and also for a sample of 85 African, Asian and western hemisphere countries during the 1984-1995 period. Using an initial regression analysis, they found some evidence that not only is there a relationship between government spending on education and school performance indicators, but also that the level of economic development has an impact on output indicators. This way, differences in economic development should be taken into account in the analysis of government spending efficiency. This is done by splitting countries into low-income and high-income (greater than US\$ 4,500 *per capita* during the 1988-91 period) groups. The categorization of the sample into groups of countries with different incomes per capita considerably alleviates the impact of differences on economic development in the efficiency analysis.

Afonso and St. Aubyn [8] also assessed the efficiency of spending on education for a sample of OECD countries. A peculiarity of their work is the use of both physical inputs (teacher-to-student ratio and hours of class attendance per year) and financial inputs (spending), since a country can be technically efficient, but inefficient if the inputs used by it are high-priced. In fact, the results turn out to be different. When physical other than financial inputs are used, Switzerland and Finland are efficient, as expected.

Afonso and St. Aubyn [9] study the efficiency of spending on education in 25 countries, mostly OECD countries, combining two different strands of the literature: efficiency of public spending and the education production function, which seeks to evaluate the determinants of quality of education using cross-country regressions.

This combination is worked out by a two-stage approach. In the first stage, the output efficiency score is estimated for each country, including only discretionary inputs, i.e., those whose amounts can be altered as desired by the countries. Two input measures are used: i) overall class time (hours per year) in public schools for students aged 12 to 14 years; ii) number of teachers per student in public and private high schools.

Socioeconomic differences (family wealth and parents' schooling) may play an important role in determining heterogeneity across countries and influence the results. It is therefore necessary to take into consideration the presence of environmental variables or factors, known as nondiscretionary inputs. Then, in the second stage, these efficiency scores are used as dependent variables in a regression analysis. The extant empirical evidence indicates that the family background variables identified by the authors who follow the education production function approach are actually highly correlated with inefficiency, i.e., they are significant environmental variables. However, these variables are fundamentally different from input variables, as their values cannot change over a short time frame (see Afonso and St. Aubyn [9]).

Herrera and Pang [10] also used a two-stage approach to assess the efficiency of spending on education for 140 countries. An interesting remark made by the authors is that, from a policy perspective, it is important to make a distinction between technical efficiency and an optimal or desirable spending level. A country could increase public spending even if it is efficient, when trying to reach a target for a given education indicator. This is the case, for example, of countries with low spending rates and low performance indicators, which are close to the origin of the efficient frontier. It is essential, then, that countries expand their operation along the efficient frontier.

Gimenez, Prior and Thieme [11] calculate efficiency scores for the schooling system of 31 countries. They also perceive that the decision to include very heterogeneous countries in the sample (OECD and developing countries) hinders comparison. When only developing countries are considered, environmental factors are essential to explain the differences in efficiency between them.

Sutherland *et al.* [12] innovate by conducting an analysis both at the country and school levels. A school-level analysis determines to what extent inefficiency varies within each country and helps identify how efficiency varies in different types of schools (small and big; public and private). Some results are particularly interesting. First, the differences in efficiency estimates are not so stark for the different school types and sizes. The average public school in the sample is slightly less efficient than the average government-dependent private school and than the average government-independent private school. Secondly, there are significant differences in efficiency estimates within certain countries. For instance, while Japan and Ireland are countries where the average level of inefficiency is low, the dispersion in efficiency is stronger for schools of the former country than for those in the latter.

Following the steps of the international literature, some studies were also carried out for Brazil. Brunet *et al.* [13] seek to relate the use of financial resources of Brazilian states and of the Federal District to the supply of products and services (efficiency) and to the obtained results (effectiveness).

Alves Júnior and Sousa [14] do the same, but they use a method in which excess inputs obtained in the first stage are estimated by a stochastic frontier, where exogenous variables are explicitly considered in order to “sort out” the inefficiency components that can be attributed to the conditions under which states operate, to statistical noise and to resource mismanagement. The aim is to control for external conditions and other factors that could favor or hinder management by states. Finally, in the third stage, corrected inputs (or pseudo-inputs) are used in a new nonparametric analysis, allowing for an effective comparison as states are assessed under the same conditions. The result for the replacement of original inputs with the corrected ones represents exclusively the level of efficiency, indicating the competence of states to manage the resources used in the education production process.

Correcting for excess consumption of resources, the results change drastically. For the group of states, the level of efficiency rises to 0.81, which means a 4.4% increase compared to the mean of 0.77 obtained in the first stage. Looking at Brazilian states individually, the most remarkable case is that of Rio de Janeiro, whose mean efficiency score for resource management was 0.62 in the first stage, which placed it as the most inefficient state in the group. After correction of inputs in the second stage, Rio de Janeiro is amongst the most efficient states, with a mean score greater than 0.88. The Federal District also increased its efficiency, from 0.71 to 0.79. On the other hand, Roraima, which had a high efficiency score in the first stage (mean of 0.87), was one of the most inefficient states in the third stage (score of 0.75).



### 3. Implementation of the empirical method

#### 3.1 Relevant issues

The review of the literature pointed out some issues that must be adequately addressed in the empirical analysis:

a) The definition of output in the field of education is complex. It is possible to adopt a narrower definition where output is measured by quantitative indicators such as enrollments, graduation rates, etc., or a broader definition based on the impact of education on welfare or on economic growth. A widely used alternative is the use of an intermediate result, measured by scores on standardized tests, which can be seen as one of the basic elements in human capital accumulation. Based on that, we chose to use IDEB, which combines the results of standardized tests and pass rates.

b) With respect to inputs, the major difficulty is to define those that are not controlled by educators and how to deal with them in the analysis. The alternative used herein was to include a socioeconomic background variable for the student directly in the production function, as a nondiscretionary input, as done by Sutherland *et al.* [12].

c) The literature postulates that environmental conditions under which countries, states and municipalities operate have a strong impact on performance. For municipalities, this factor is even more important. Therefore, it is necessary to somehow take into account that different local governments are faced with distinct social and economic conditions. In other words, municipalities deal with different production opportunities or make choices based on different sets of possible input-output combinations. The technologies are different simply because there are differences in physical and human capital stock (number of computers and teachers' qualifications), in economic infrastructure and in other characteristics of the environment where production takes place. So, it is important to have the results for different groups of municipalities.

#### 3.2 Data and methodology

In addition to expenditures per student, according to SIOPE (Information System on Public Spending on Education), the following inputs are considered: mothers' (or legal representatives') schooling for students aged up to 20 years who attend elementary or middle school at a public institution (better *proxy* available in the Census 2010 microdata).

The Brazilian Education Quality Index (IDEB) will be used as output measure. Created in 2005 to measure the quality of public elementary/middle school education, IDEB indicates the pass rates and performance of students on *Prova Brasil*, standardized tests applied at the end of each cycle.

It is worth mentioning that IDEB includes targets to be met on a yearly basis such that education levels comparable to those of developed countries will have been achieved by 2021, providing us with a clear objective in terms of output.

An important issue that underlies this discussion is what the minimum amount of spending would be in order to meet the targets established by IDEB for each municipality. If the minimum amount of spending for targets to be met is lower than the current effective spending, this may

indicate that the municipality does not necessarily need more resources. In this case, it is possible to assess to what extent resources have been wasted.

The methodology therefore includes two steps. The first one consists in calculating the waste of resources for each municipality. The difference between the current effective spending and wasted resources corresponds to the spending that should be necessary if the municipality were efficient. This spending will be referred to as efficient spending. The second step consists in calculating the minimum amount of spending necessary to meet the established 2021 IDEB targets for each municipality. Finally, the current effective spending will be compared with the minimum amount of spending necessary to meet the target. If the former is greater than the latter, the municipality will not need an increase in resources.

To calculate the waste of resources in the field of education for each municipality, we use directional distance functions. In traditional efficiency measures, all inputs are reduced by the same factor, that is, all inputs are regarded as discretionary. In the directional distance functions approach, it is possible to specify which inputs are discretionary or also the extent to which inputs can be reduced.

Our model is based on a production process with  $n$  outputs and  $m$  inputs, where  $y \in \mathbb{R}_+^n$  is the vector of quantities of output and  $x \in \mathbb{R}_+^m$  is the vector of quantities of inputs. An activity is expressed by  $(x, y) \in P \subset \mathbb{R}_+^m \times \mathbb{R}_+^n$  and contains the input-output combination of a given DMU which, in our case, is a municipality. The set formed by all feasible activities is called production possibility set ( $T$ ).

We define  $d \in \mathbb{R}_+^m$  as a vector that determines an arbitrary direction from which inputs can be reduced. For instance, in the case where  $m = 3$ , vector  $d = (1, 0, 0)$  defines the direction of  $x_1$  to reduce the inputs, which leads to the understanding that only input  $x_1$  is discretionary.

Given an activity  $(x^0, y^0)$ , we define the directional distance function or excess function as:

$$e = e(x^0, y^0, T, d) = \max\{e \in \mathbb{R}_+ \mid (x^0 - ed, y) \in T\}$$

Excess  $e(x^0, y^0, T, d)$  corresponds to the number of units in the input basket, defined by vector  $d$ , used in excess of the efficient spending to produce  $y^0$ , such that the resulting activity belongs to the production possibility set. Hence,  $e$  can be interpreted as waste of inputs. It should be noted that this approach differs from the traditional radial construction, as it reduces only the discretionary inputs.

The additive variant of the DEA model (see Ray [15] pp.120, Bogetoft and Otto [16] pp. 121) can be used to calculate and consists of the following linear problem:

$$\begin{aligned} & \text{Max}_{e, \lambda_1, \dots, \lambda_k} e \\ & \text{s. t. } x^0 - ed \geq \sum_{k=1}^K \lambda_k X^k \\ & \quad y^0 \leq \sum_{k=1}^K \lambda_k Y^k \\ & \quad \lambda \in \Lambda^k(\gamma) \end{aligned}$$

Where  $\lambda \in \Lambda^k(\gamma)$  is a constraint imposed on weights  $\lambda$ , which defines the returns to scale. In the specific case, we have:

$X^k$ : a  $K \times 2$  input matrix, in our case:  $X = (\text{Spending}_j, \text{MotherSchooling}_j)_{j=1, \dots, K}$

$Y^k$ : a  $K \times 1$  output matrix, in our case:  $Y = (\text{Ideb}_j)_{j=1, \dots, K}$

$d$ : Direction vector. In this case, as  $X = (\text{Spending}, \text{MotherSchooling})$ , to calculate wasted resources relative only to spending (discretionary input in the short run), we use vector  $d = (1, 0)$

The result of  $f$  gives the directional waste of spending. For the sake of comparison, we also use the traditional DEA model.

The identification of the minimum amount of spending necessary to achieve the IDEB target for 2021 would be trivial if we were working with stochastic frontiers. The production function properties (Coelli *et al.* [17], pp. 12), in special monotonicity, would suffice to guarantee the invertibility of the production function<sup>1</sup>.

In this case, the minimum amount of spending is calculated as:

$$\text{MinSpending}_j = f^{-1}(\text{IdebTarget}_j, \overline{\text{MotherSchooling}_j})$$

Where  $f(\cdot, \overline{\text{MotherSchooling}_j}): \mathbb{R} \rightarrow \mathbb{R}$  is the production function after determination of schooling,  $f^{-1}(\cdot, \overline{\text{MotherSchooling}_j}): \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is its inverse function and  $\text{MinSpending}_j$  is the minimum amount of spending necessary for the DMU  $j$  to meet the IDEB target.

Because we are working with DEA, it was necessary to find a procedure that allowed calculating the minimum amount of spending necessary to meet the IDEB target, after determination of schooling. The proposed linear problem takes the following form:

$$\begin{aligned} & \text{Min}_{\text{MinSpending}_i, \lambda_1, \dots, \lambda_k} \text{MinSpending}_i \\ \text{s. t. } & \sum_{j=1}^K \lambda_j \text{Ideb}_j \geq \text{IdebTarget}_i \\ & \sum_{j=1}^K \lambda_j \text{MotherSchooling}_j \leq \text{MotherSchooling}_i \\ & \sum_{j=1}^K \lambda_j \text{Spending}_j \leq \text{MinSpending}_i \end{aligned}$$

The solution to this linear problem yields activity  $(\text{MinSpending}_i, \text{MotherSchooling}_j, \text{IdebTarget}_j)$ . The  $\text{MinSpending}_i$  value is the minimum amount of spending necessary to meet the IDEB target. In the VRS case, it is necessary to include the constraint  $\sum_{j=1}^K \lambda_j = 1$ . The proof that this activity belongs to the production possibility set is shown in Appendix A.

<sup>1</sup> Here, we refer to the mathematical concept of inverse function and not to the inversion of the production process.

## 4. Results

As pointed out earlier, the technical efficiencies calculated by the traditional DEA model will also be shown for the sake of comparison. Table 1 displays the estimated scores by assuming constant returns to scale.

**Table 1 - Estimation of DEA Technical Efficiencies (Constant Returns to Scale)**

	Number of municipalities	Mean	Minimum	Maximum
Up to 50,000 inhabitants	4,341	0.503	0.172	1.000
50,000 to 100,000 inhabitants	312	0.567	0.297	1.000
100,000 to 500,000 inhabitants	236	0.682	0.433	1.000
Over 500,000 inhabitants	32	0.812	0.629	1.000

Note: Data compiled by the authors.

As can be seen, efficiency increases with the size of municipalities. Thus, while smaller municipalities (up to 50,000 inhabitants) could reduce their inputs by nearly 50% and continue to have the same IDEB result, larger municipalities (with over 500,000 inhabitants) can reduce their inputs by almost 20%.

In any case, the waste of resources by larger municipalities is still significant. The variance of inefficiency is in line with that of the mean. Among smaller municipalities, the most inefficient one has an efficiency score of 0.172, whereas among larger municipalities, the most inefficient one has an efficiency score of 0.629. This stark difference in the efficiency of small municipalities leads to a lower mean score than the lowest score of larger municipalities. The mean efficiency score is 0.503 for municipalities with up to 50,000 inhabitants and 0.567 for municipalities with 50,000 to 100,000 inhabitants. However, the minimum score for larger municipalities is 0.629.

Special attention should be paid to the fact that the scores were computed by constructing efficiency frontiers for each group, indicating heterogeneous scores for each municipality and variations across the groups. Therefore, the comparison between groups is not strictly accurate since it would be necessary to use the metafrontier approach, which allows comparing the technical efficiencies of municipalities across different groups that might not have the same technology (see Battese and Rao [18] and Battese, Rao and O'Donnell [19]). Table 2 shows the results for the traditional DEA model, assuming variable returns to scale.

**Table 2 - Estimations of DEA Technical Efficiencies (Variable Returns to Scale)**

	Number of municipalities	Mean	Minimum	Maximum
Up to 50,000 inhabitants	4,341	0.564	0.196	1.000
50,000 to 100,000 inhabitants	312	0.655	0.307	1.000
100,000 to 500,000 inhabitants	236	0.719	0.447	1.000
Over 500,000 inhabitants	32	0.868	0.675	1.000

Note: Data compiled by the authors.

The results follow the same pattern verified when constant returns to scale were used: larger municipalities are, on average, more efficient, the variance of efficiency scores of smaller municipalities is higher and the efficiency scores of smaller municipalities are lower than the lowest efficiency score of municipalities with over 500,000 inhabitants.

Table 3 shows the estimates for wasted resources, i.e., how much more municipalities spend compared to the necessary resources to achieve efficiently the IDEB they managed to realize. Wasted resource is calculated as the sum of expenditures of each municipality in that group multiplied by the complement of the average efficiency score of each municipality. For instance, if the efficiency score of a municipality is 0.8 and it spends R\$100, the amount of wasted resources is R\$20. This calculation is repeated for each municipality and is then aggregated to yield the amount of wasted resources for each group. Efficient spending corresponds to how much the municipality should spend to have the same output level. In the previous example, it would be R\$80, i.e., it is the difference between efficient spending and wasted resources. Again, the calculation is made for each municipality of that group and is then added.

**Table 3 - Estimations of Wasted Resources and Efficient Spending (DEA)**

	Wasted resources DEA - CRS	Efficient spending DEA - CRS	Wasted resources DEA - VRS	Efficient spending DEA - VRS
Up to 50,000 inhabitants	R\$ 10,668 billion	R\$ 11,467 bil- lion	R\$ 9,108 billion	R\$ 12,965 billion
50,000 to 100,000 inhabitants	R\$ 3,145 billion	R\$ 3,735 bil- lion	R\$ 2,607 billion	R\$ 4,274 bil- lion
100,000 to 500,000 inhabitants	R\$ 5,217 billion	R\$ 8,389 bil- lion	R\$ 4,773 billion	R\$ 8,833 bil- lion
Over 500,000 inhabitants	R\$ 3,425 billion	R\$ 8,759 bil- lion	R\$ 3,019 billion	R\$ 9,165 bil- lion
Total	R\$ 22,455 billion	R\$ 32,350 bil- lion	R\$ 19,506 billion	R\$ 35,237 billion

Note: Data compiled by the authors.

Tables 4 and 5 show the effective spending, the estimates of wasted resources by the MEA model, as well as the minimum amount of spending necessary to meet the target. The MEA result provides the directional waste of spending, assuming that mother's schooling cannot vary in the short run. Again, efficient spending corresponds to current effective spending minus wasted resources. The results are impressive. The wasted resources of municipalities in group 1, group 2, group 3 and group 4 account for 54.9%, 42.5%, 39.6% and 44.6% of the overall effective spending, respectively, when constant returns to scale are assumed. In the case of variable returns to scale, wasted resources account for a smaller proportion of effective spending, but even so, the amount is significant: 46.2% for group 1, 35.6% for group 2, 36.2% for group 3 and 35.7% for group 4.

For any group, actual spending is much higher than the minimum amount of spending necessary to meet the target.

**Table 4 - Spending, Efficient Spending, Wasted Resources and Minimum Amount of Spending to Meet the Target (Constant Returns to Scale - CRS)**

	Spending today	Efficient Spending MEA - CRS	Wasted resources MEA - CRS	Minimum Spending to Meet the 2021 Target
Up to 50,000 inhabitants	R\$ 22,134 billion	R\$ 9,974 billion	R\$ 12,160 billion	R\$ 13,461 billion
50,000 to 100,000 inhabitants	R\$ 6,880 billion	R\$ 3,958 billion	R\$ 2,922 billion	R\$ 4,307 billion
100,000 to 500,000 inhabitants	R\$ 13,606 billion	R\$ 8,223 billion	R\$ 5,383 billion	R\$ 6,975 billion
Over 500,000 inhabitants	R\$ 12,184 billion	R\$ 6,753 billion	R\$ 5,432 billion	R\$ 4,499 billion
Total	R\$ 54,805 billion	R\$ 28,908 billion	R\$ 25,897 billion	R\$ 29,241 billion

Note: Data compiled by the authors.

**Table 5 - Spending, Efficient Spending, Wasted Resources and Minimum Amount of Spending to Meet the Target (Variable Returns to Scale - VRS)**

	Spending today	Efficient Spending MEA - VRS	Wasted Resources MEA - VRS	Minimum Spending to Meet the 2021 Target
Up to 50,000 inhabitants	R\$ 22,134 billion	R\$ 11,833 billion	R\$ 10,239 billion	R\$ 13,708 billion
50,000 to 100,000 inhabitants	R\$ 6,880 billion	R\$ 4,432 billion	R\$ 2,448 billion	R\$ 4,419 billion
100,000 to 500,000 inhabitants	R\$ 13,606 billion	R\$ 8,685 billion	R\$ 4,921 billion	R\$ 7,177 billion
Over 500,000 inhabitants	R\$ 12,184 billion	R\$ 7,838 billion	R\$ 4,346 billion	R\$ 4,647 billion
Total	R\$ 54,805 billion	R\$ 32,789 billion	R\$ 21,967 billion	R\$ 29,951 billion

Note: Data compiled by the authors.

Table 6 shows the difference in the estimated amount of wasted resources using the traditional DEA model and MEA. The first column displays the results for the model with constant returns to scale while the second column presents the results for the model with variable returns to scale. Except for the group with 50,000 to 100,000 inhabitants, the difference is negative, indicating that the amount of wasted resources estimated by MEA is greater than that calculated with DEA.

**Table 6 - Difference in the Estimated Amount of Wasted Resources (DEA vs. MEA)**

	Constant Returns to Scale	Variable Returns to Scale
Up to 50,000 inhabitants	- R\$ 1,492 billion	- R\$ 1,131 billion
50,000 to 100,000 inhabitants	R\$ 223 million	R\$ 159 million
100,000 to 500,000 inhabitants	- R\$ 166 million	- R\$ 148 million
Over 500,000 inhabitants	- R\$ 2,007 billion	- R\$ 1,327 billion
Total	- R\$ 3,442 billion	- R\$ 2,447 billion

Note: Data compiled by the authors.

Finally, we could say that no more resources are necessary because the targets for 2021 were not set so high. Some simulations were then carried out, where the fixed targets were increased by 2.5%, 5% and 7.5%. The results for all municipalities are shown in Tables 7 and 8 for constant and variable returns to scale and indicate that, while targets were more strict, the amount of resources wasted today would be more than enough to cover the minimum amount required for these tougher targets to be met.

**Table 7 - Simulation with more strict targets- (Constant Returns to Scale - CRS). in R\$ billion.**

	Municipalities	Spending today	Spending target CRS
Spending on the 2021 target	4,955	54,805	29,241
Spending on the 2021 target + 2.5%	4,955	54,805	30,096
Spending on the 2021 target + 5.0%	4,955	54,805	30,962
Spending on the 2021 target + 7.5%	4,954	54,801	31,836
Spending on the 2021 target + 10%	4,951	54,782	32,693

Note: Data compiled by the authors.

**Table 8 - Simulation with more strict targets - (Variable Returns to Scale - VRS). in R\$ billion.**

	Municipalities	Spending today	Spending Target VRS
Spending on the 2021 target	4,942	54,743	29,951
Spending on the 2021 target + 2.5%	4,934	54,713	30,770
Spending on the 2021 target + 5.0%	4,923	54,677	31,685
Spending on the 2021 target + 7.5%	4,911	54,625	32,680
Spending on the 2021 target + 10%	4,884	54,484	33,686

Note: Data compiled by the authors.

## 5. Conclusions

The aim of this paper is to assess whether the resources municipalities allocate to education are enough to meet the targets set for 2021. To do that, we use directional distance functions, in which we specify only the proportion by which discretionary inputs can be reduced, instead of proportionally reducing all inputs, as done in traditional distance functions.

The spending of municipalities on elementary/middle school education and mother's schooling are used as inputs, but we assume that the latter is a fixed input in the short run that lies outside the immediate discretionary power of a municipality. Consequently, we seek to assess only the impact of improvements in the discretionary (controllable) domain, i.e., in spending itself.

After measuring the distance of each municipality from the education production frontier, that is, the waste of resources, we solved a linear programming problem similar to that of the DEA model, whose result is the minimum amount of spending necessary for a municipality to reach its 2021 IDEB target.

The results indicate that the amount of wasted resources is significant for any group of municipalities, defined by its population size. For the whole group of municipalities, wasted resources account for 47.3% and 40.1% of the overall effective spending when constant returns to scale and variable returns to scale are used, respectively.

Effective spending is much higher than the minimum amount of spending necessary to meet the targets. Even when we carry out simulations with more strict targets, we clearly notice that the lack of resources is not a constraint.

## 6. References

[1] Menezes-Filho N, Pazello E. Do teachers' wages matter for proficiency? Evidence from a funding reform in Brazil. *Economics of Education Review* 2007; 26:660-672.

[2] Mendes M, Miranda RB, Cossio F. Transferências intergovernamentais no Brasil: diagnóstico e proposta de reforma. Consultoria Legislativa do Senado Federal, Texto para Discussão 2008; 40.

[3] Hanushek E, Kimko, D. Schooling, labor force quality, and economic growth. *American Economic Review* 2000; 90:1184-1208.

[4] Hanushek E, Luque, J. Efficiency and equity in schools around the world. *Economics of Education Review* 2003; 22: 481-502.

[5] Amaral LFLE, Menezes Filho NA. A relação entre gastos e desempenho educacional. In XXXVI Encontro Nacional de Economia, 2008, Salvador - Bahia. Anais do XXXVI Encontro Nacional de Economia, 2008.

[6] Clements B. How efficient is public spending in Europe? *European Review of Economics and Finance* 2002; 1: 3-26, 2002.

[7] Gupta S, Verhoeven M, Tiongson ER. The effectiveness of government spending on education and health care in developing and transition economies. *European Journal of Political Economy* 2002; 18: 717-737.



- [8] Afonso A, St Aubyn M. Non-parametric approaches to education and health efficiency in OECD countries. *Journal of Applied Economics* 2005; VIII(2): 227-246.
- [9] Afonso A, St Aubyn M. Cross-country efficiency of secondary education provision: A semi-parametric analysis with non-discretionary inputs. *Economic Modeling* 2006; 23 (3): 476-491.
- [10] Herrera S, Pang G. Efficiency of public spending in developing countries. *World Bank Policy Research Working Paper* 3645 2005.
- [11] Gimenez V, Prior D, Thieme C. Technical efficiency, managerial efficiency and objective setting in the educational system: an international comparison. *Journal of the Operational Research Society* 2007; 58: 996-1007.
- [12] Sutherland D, Price R, Joumard L, Nicq C. Performance indicators for public spending efficiency in primary and secondary education. *OECD Economics Department Working Paper* n. 546 2007.
- [13] Brunet JFG et al. Estados comparados por funções do orçamento: uma avaliação da eficiência e efetividade dos gastos públicos estaduais. *Prêmio IPEA-Caixa, Menção Honrosa I*, 2006.
- [14] Alves Jr JA, Souza MCS. Eficiência na aplicação de recursos educacionais em presença de condicionantes exógenos e efeitos aleatórios: uma abordagem em três estágios. In *33º Encontro Brasileiro de Econometria*, 2011, Foz do Iguaçu - Paraná. *Anais do 33º Encontro Brasileiro de Econometria*, 2011.
- [15] Ray SC. *Data Envelopment Analysis: Theory and techniques for economics and operations research*. New York: Cambridge University Press; 2004
- [16] Bogetoft P, Otto L. *Benchmarking with DEA, SFA and R*. New York: Springer, Verlag; 2011.
- [17] Coelli TJ, Rao DSP, O'Donnell CJ, Battese GE. *An introduction to efficiency and productivity analysis*. 2nd ed. New York: Springer Science + Business Media; 2005.
- [18] Battese GE, Rao DSP. Technology gap, efficiency, and a stochastic metafrontier function. *International Journal of Business and Economics* 2002; 1 (2): 87-93.
- [19] Battese GE, Rao DSP, O'Donnell CJ. A metafrontier production function for estimation of technical efficiencies and technology gaps for firms operating under different technologies. *Journal of Productivity Analysis* 2004; 21: 91-103.
- [20] Cooper WW, Seiford LM, Tone K. *Data envelopment analysis: a comprehensive text with models, applications, references and DEA-Solver software*. 2nd ed. New York: Springer Science + Business Media; 2007.

## Appendix

**Proposition:** The activity defined by  $(\text{MinSpending}_i, \text{MotherSchooling}_i, \text{IDEBTarget}_i)$  belongs to the production possibility set  $(T)$ .

**Proof:**

The production possibility set  $(T)$  has the following properties (see Cooper, Seiford and Tone [20]):

I - if  $(x, y) \in P$ , any semipositive activity  $(\bar{x}, \bar{y})$  with  $\bar{x} \geq x$  and  $\bar{y} \leq y$  also belongs to the production possibility set. In other words, any activity using an input that is no less than  $x$  and an output that is no greater than  $y$  is feasible.

II - Any semipositive linear combination of activities in  $P$  belongs to  $P$ . That is,  $(\sum_j \lambda_j x_j, \sum_j \lambda_j y_j) \in P$  if  $(x_j, y_j) \in P$ , for all  $j$ .

Based on Property II, given  $(\text{MinSpending}_j, \text{MotherSchooling}_j, \text{IDEBTarget}_j) \in P$ , with  $j = 1 \dots N$ , we have that activity  $(\sum_j \lambda_j \text{Spending}_j, \sum_j \lambda_j \text{MotherSchooling}_j, \sum_j \lambda_j \text{IDEB}_j)$  also belongs to the production possibility set.

If  $\text{MinSpending}_i \geq \sum_j \lambda_j \text{Spending}_j$ ,  $\text{MotherSchooling}_i \geq \sum_j \lambda_j \text{Schooling}_j$ , and  $\text{IdebTarget}_i \leq \sum_j \lambda_j \text{ideb}_j$ , according to Property I, activity  $\text{IdebTarget}_i \leq \sum_j \lambda_j \text{ideb}_j$  also belongs to the production possibility set.

Since these are restraints of the linear programming model, we have that  $(\text{MinSpending}_i, \text{MotherSchooling}_i, \text{IDEBTarget}_i) \in T$ .

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